A COMPARATIVE STUDY OF SOME METHODS USED IN THE DETERMINATION OF ECONOMIC GROWTH

[1]Kayode Olarewaju Joseph, [2]Ikpotokin Festus O. and [3]Omosuyi I.O
[1&3]Department of Mathematics and Statistics, Rufus Giwa Polytechnic, Owo, Ondo State, Nigeria
[2]Department of Computer Science, Ambrose Alli University, Ekpoma, Edo State, Nigeria
E-mail: kdolarewaju@gmail.com,

Abstract
This work is based on a comparative study of some methods used in the determination of economic growth. The data for this research work were secondary data and were obtained from the Central Bank of Nigeria. The estimators of Ordinary Least Squares (OLS), Maximum Likelihood Estimator (MLE) and Seemingly Unrelated Regression Estimator (SURE) were used and compared. The result of the analysis showed that the co-efficients for all regressors were found to be statistically significant under the three estimators considered, since the probability values of the regressors coefficient are less than the level of significant 0.05 as can be seen in Table 4.6. For instance, the coefficient for INTR (0.0129, 0.0129, and 0.0125) has probability values of 0.0055, 0.0052, and 0.0055, respectively, which are values less than 0.05 for the three approaches (OLS, MLE, and SUR). This indicates that there is a statistically significant relationship between ER and INTR, such that an increase of one unit in the INTR causes an increase in ER of around 1.3%. Also, MLE has the minimum standard error for both the Gross Domestic Product (GDP) and External Reserve (ER) estimations, which indicate the MLE to be the most efficient among the three methods. For instance, when the standard error for IOP was considered, it was observed that the GDP for MLE (0.0036) compared with OLS (0.0038) and SUR (0.0038) happened to be the smallest value. In the same vain, ER for ML (0.0037) compared with OLS (0.0039) and SUR (0.0039) equally has the smallest value.
Keywords: Economic Growth, Estimators, Multicollinearity, Efficiency,

1. Introduction

Economic growth is an increase in an economy's total, or per-capital, or output, which is often quantified by a rise in real Gross National Product and brought on by a rise in the availability of production resources or their productivity. Fiscal and monetary policy are the two main macroeconomic instruments for achieving sustainable growth.

According to Bodunrin (2016), the economy has not yet felt the effects of the monetary authority's interference despite years of deficit budgets and interest rate manipulation of the money supply and demand. Money supply has increased quickly over time, relative to government spending, despite the fact that government spending has been unpredictable and actual economic growth has been slow. Bayo (2020) carried out a study to determine the impact of monetary policy measures on economic growth and found out that, among other things, monetary policy is not the sole determinant of economic growth, but that cash ratio was found to have a significant impact on economic growth.

The factors that affect economic growth are getting more and more attention in both theoretical and empirical research. One reason for this is that a country's long-term development and stability depend on its economy growing steadily over time (Udeaja & Onyebuchi, 2015). Every year, rising inflation, high unemployment, low production, and dwindling foreign reserves lead to unstable exchange rates, especially when the price of oil keeps falling (Central Bank of Nigeria, 2015). Nigeria's Gross Domestic Product (GDP) dropped from 553 billion USD in 2014 to 479 billion USD in 2015; in the first quarter of 2015, more than 17.7 million people aged 15 to 65 were either unemployed or underemployed; and inflation rose from 8% to 9.6%. (The National Bureau of Statistics, 2015).

A country's economic progress is measured by a rise in its Gross Domestic Product (GDP). According to Onuoha, Ibe, Njoku & Onuoha (2015), GDP is one of the major metrics used to assess a country's health. Some of the economic elements that commonly affect a country's GDP and Foreign Reserve (FR) are: oil income, money supply, agricultural export, foreign private investment, federal government spending,

interest rate, exchange rate, inflation, among others. This study, therefore, used ordinary least squares (OLS), seemingly unrelated regression estimation (SURE) and maximum likelihood estimation (MLE), to estimate the system of economic data, identify the statistical contributors to Nigeria's Gross Domestic Product (GDP) and External Reserves (ER), and determine the estimation techniques that allow for both types of correlation and produce more accurate estimates of the regression coefficients than ordinal least squares.

2.0 Materials and Methods

2.1 Ordinary Least Square (OLS)

Estimating the conditional means function may be accomplished with the help of a number of linear regression models. One of these models is the ordinary least squares (OLS) technique. Estimating the OLS model's coefficients requires reducing the sum of the squared residuals in order to get the best results. Because of this, the equation for OLS may be written as:

$$y = \sum_{i=1}^{n} \beta^i x_i + \varepsilon \quad (1)$$

OLS estimation for each equation yields a consistent estimator of β, but the optimal estimator is the GLS estimator.

$$\hat{\beta}_{GLS} = \left\{X'(\Sigma^{-1} \otimes I_N)X\right\}^{-1}\left\{X'(\Sigma^{-1} \otimes I_N)y\right\} \quad (2)$$

With $\text{var}(\hat{\beta}) = \left\{X'(\Sigma^{-1} \otimes I_N)X\right\}^{-1}$ (3)

2.2 Seemingly Unrelated Regression Estimate (SURE)

SUR model is a system of linear equations with errors that are correlated across equations for a given individual but are uncorrelated across individuals. The model consists of $j = 1,2,...,m$ linear regression equations for $i = 1,2,...,N$ individuals *j*th equation for individual *I* is.

$$y_{ij} = x_j'\beta_j + u_{ij} \quad (4)$$

We can stack the m equations into an SUR model:

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_m \end{bmatrix} = \begin{bmatrix} X_1 & 0\cdots & 0 \\ . & \ddots & \vdots \\ 0 & \cdots & X_m \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_m \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ . \\ u_m \end{bmatrix} \quad (5)$$

For a given individual, the errors are correlated across equations,

$$E\left(u_{ij}u_{ij'}|X\right) = \sigma_{jj'} \neq 0 \text{ where } j \neq j' \quad (6)$$

2.3 Maximum Likelihood Estimation (MLE)

Define a $nt \times nt$ matrix L which can be obtained from the identity matrix I_{nt}, by exchanging its rows such that: $LL' = L'L = I_{nt}$ (7)

And $$LY = L\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_t \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad (8)$$

Where $Y_j = (y_{1j}, y_{2j},..., y_{nj})$ is $n \times 1$ observation vector at time j and $y_i = (y_{i1}, y_{i2},..., y_{it})'$ is a $t \times 1$ observation vector of the ith subject. Pre-multiplying L on both side of $Y = X\beta + \varepsilon$ we have

$$LY = LX\beta + L\varepsilon \equiv y = Z\beta + e \quad (9)$$

2.4 Model Specification

For the purpose of this study, two models were considered;

Model 1

$$GDP = IOP_i + INFR_i + FDI_i + \varepsilon_i \quad (10)$$

Model 2

$$ER = EXR_i + INTR_i + IOP_i + \varepsilon_i \quad (11)$$

3.0 Summary of Dataset

	N	MEAN	SD	MEDIAN	MIN	MAX	SE
GDP	36	136220.5	169645.7	45262.01	15789	568499	28274.28
ER	36	16582.68	18169.45	7356.82	932.99	53599.28	3028.24
EXR	36	76.47	71.95	57.2	0.62	253.49	11.99
INTR	36	-0.11	16.59	3.23	-43.57	25.28	2.77
IOP	36	40.33	30.19	27.85	12.28	109.45	5.03
INFR	36	19.6	17.69	12.55	5.40	72.80	2.95
FDI	36	485.01	524.49	260.43	-27.00	1601.23	87.41

3.1 Description of Economic Variables

Figures 1 to 6 show the trends of Nigeria's Gross Domestic Product (GDP), External Reserves (ER), Exchange Rate (EXR), Interest Rate (INTR), International Oil Price (IOP), Inflation Rate (INFR) and Foreign Direct Investment (FDI) for the period under review.

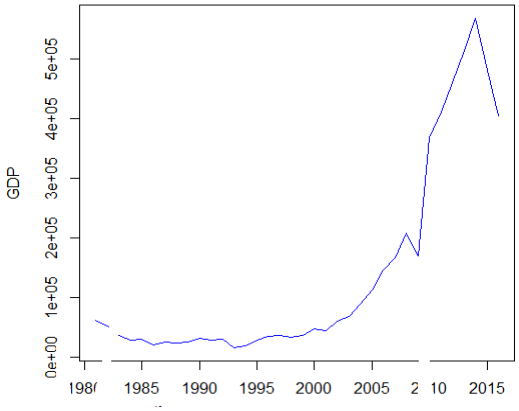

Figure 1: Nigeria Gross Domestic Product (in millions)

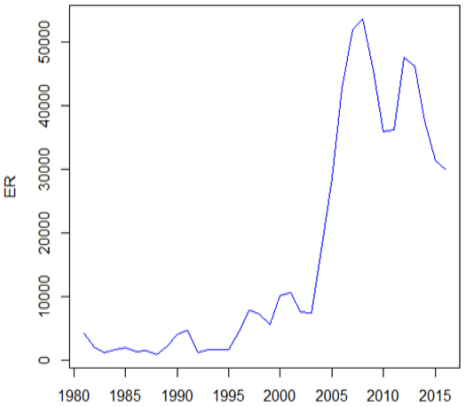

Figure 2: Nigeria External Reserves (in Million Dollar)

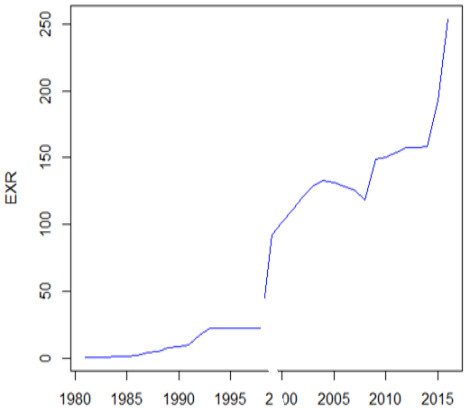

Figure 3: Nigeria's Exchange Rate (Naira/Dollar)

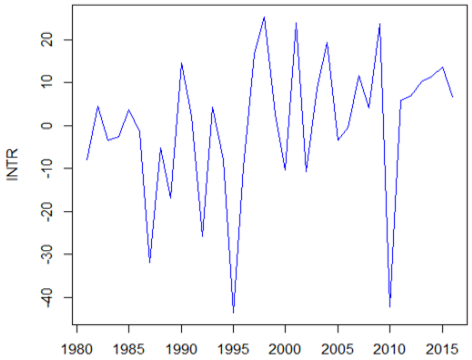

Figure 4: Nigeria's Interest Rate (%)

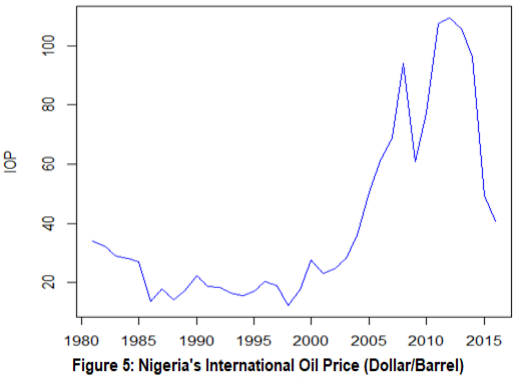

Figure 5: Nigeria's International Oil Price (Dollar/Barrel)

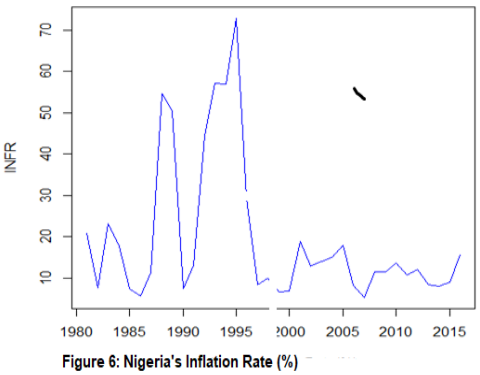

Figure 6: Nigeria's Inflation Rate (%)

3.2 Diagnostic Test

3.2.1 Normality Test: Jacques-Bera Test

	GDP	ER	EXR	INTR	IOP	INFR	FDI
JB Statistics	12.321	5.4404	2.6139	6.2001	8.6242	20.072	5.5003
JB P-Value	0.0021	0.0659	0.2706	0.0451	0.01341	0.000	0.0639

Significant level=0.05

When examining a continuous variable using the distribution platform, the Jacques-Bera Test for Normality is accessible. The data are regularly distributed, and this is the null hypothesis for this test. The P values for GDP, INTR, IOP, and INFR are less than the 0.05 significant threshold when the JB findings are examined. Thus, we reject the null hypothesis that says "the data are normally distributed", this means that the GDP, INTR, IOP and INFR are different from normal distribution. However, the P value of ER, EXR and FDI are greater than the chosen significance level 0.05. Therefore, we have no sufficient reason to reject the null hypothesis that says "the data are normally distributed". This means that the ER, EXR and FDI belong to normal distribution.

3.2.2 Homoscedasticity Test (Breusch-Pagan)

Equation 1:GDP		Equation 2: External Reserves	
BP Statistics	BP P-Valu	BP Statistics	BP P-Value
20.651	0.0001	10.387	0.0156

The Breusch-Pagan test's null hypothesis assumed that the variance was always the same. The estimated BP-statistics for the GDP and ER models were 20,651 and 10,387, respectively, with a probability value of 0.0001 and 0.0156 for each. So, the null hypothesis is rejected at the 0.05 level of significance for both models, and it can be said that the residuals are different.

3.2.3 Variance Inflation Factors of the Regressors

Equation 1			Equation 2		
Variables	**VIF**	**1/VIF**	**Variables**	**VIF**	**1/VIF**
EXR	1.926	0.519	IOP	2.460	0.407
INTR	1.090	0.918	INFR	1.212	0.825
IOP	1.829	0.547	FDI	2.154	0.464

The Jarque-Bera test for the LGDP (4,1321) and LER (3,3734) was unable to reject the null hypothesis of normality with probabilities of 0.1267 and 0.1867, respectively. Similarly, the computed BP-statistics for the GDP and ER models were 11.042 and 1.3425, respectively, with corresponding probability values of 0.0115 and 0.7191. Therefore, the null hypothesis could not be rejected at the 0.05 level of significance for the ER model, and it was decided that the residuals of the ER model are homogenous. However, the null hypothesis was rejected for the GDP model.

3.3 Model of LGDP and LER via OLS, MLE and SUR Methods

	OLS		ML		SUR	
	GDP	ER	GDP	ER	GDP	ER
Constant	10.1436 (0.1790) 0.0000*	7.3613 (0.1484) 0.0000*	10.1436 (0.1687) 0.0000*	7.3613 (0.1400) 0.0000*	10.1532 (0.1789) 0.0000*	7.3559 (0.1484) 0.0000*
EXR		0.0109 (0.0017) 0.0000*		0.0109 (0.0016) 0.0000*		0.0112 (0.0017) 0.0000*
INTR		0.0129 (0.0055) 0.0244*		0.0129 (0.0052) 0.0122*		0.0125 (0.0055) 0.0286*
IOP	0.0217 (0.0038) 0.0000*	0.0180 (0.0039) 0.0000*	0.0217 (0.0036) 0.0000*	0.0180 (0.0037) 0.0000*	0.0216 (0.0038) 0.0000*	0.0175 (0.0039) 0.0000*
INFR	-0.0115 (0.0046) 0.0168*		-0.0115 (0.0443) 0.0074*		-0.0118 (0.0046) 0.0146*	
FDI	0.0007 (0.0002) 0.0011*		0.0007 (0.0002) 0.0001*		0.0007 0.0002 0.0011*	
R^2	0.8661	0.8733			0.8660	0.8731

Standard errors are in parentheses and probability values are with asterisk

Covariance Matrix of Residuals of GDP and ER Model (Equation)

$$\begin{bmatrix} 0.1877 & -0.0169 \\ -0.0169 & 0.2659 \end{bmatrix}$$

Correlation of the Residuals

$$\begin{bmatrix} 1.0000 & -0.0756 \\ -0.0756 & 1.0000 \end{bmatrix}$$

The model for Nigeria's GDP and ER using the OLS, MLE, and SUR techniques of estimation is shown in Table 4.6 above. The coefficient for EXR (0.0109, 0.019, and 0.0112) has a probability value of 0.0000, which is less than 0.05 under the three techniques (OLS, MLE, and SUR), respectively, when the results are examined. According to the null hypothesis, EXR has no appreciable impact on Nigeria's ER. As a result, the 5% threshold of significance was used to reject the null hypothesis, and it was determined that EXR is positively significant using all three approaches. This indicates that an increase in the coefficient of EXR results in an approximately 1.1% rise in ER.

Additionally, the coefficient for INTR (0.0129, 0.0129, and 0.0125) has probability values of 0.0055, 0.0052, and 0.0055, respectively, which are values less than 0.05 for the three approaches (OLS, MLE, and SUR). The null

hypothesis was therefore rejected at the 5% level of significance, indicating that there is a statistically significant relationship between ER and INTR, such that an increase of one unit in the INTR coefficient causes an increase in ER of around 1.3%.

4. Conclusion and Recommendation

The result of the analysis showed that the:

(1) Trend of Economic Growth was determined. This can be seen in Figure 1 to Figure 6.
(2) Co-efficient for all regressors were found to be statistically significant under the three estimators considered.
(3) Co-efficient variation revealed that the regressors (IOP, INFR, and FDI) can explain 86.6% of variation in GDP, while the regressors (EXR, INFR, and IOP) explain 87.3% of variation in ER.
(4) MLE has the minimum standard error for both GDP and ER models which confirmed the MLE estimator to be the most efficient among the three methods.
(5) SUR estimation showed that the covariance matrix of the residual and correlation of the residual of the two models (GDP and ER model) is negative (-0.0756).
(6) Breusch-Pagan test for error independence showed a chi-square test statistic of 13.431 and has a probability value 0.0038, which is less than 0.05. This is an indication that there is statistically significant correlation between the errors in the two equations (but correlation of -0.0756 is not particularly strong).

The purpose of the study was to determine the effect that macroeconomic factors have on the economic growth of Nigeria. The study used Nigeria's Gross Domestic Product (GDP) and External Reserves (ER) as proxies for economic growth, while the macroeconomic factors included Exchange Rate, Interest Rate, International Oil Price, Inflation Rate, and Foreign Direct Investment as independent variables. The GDP and ER models were developed in such a way that the EXR, INTR, and IOP variables are regressed on the external reserve variables, and the IOP, INFR, and FDI variables are also regressed on the GDP variables. These regressions were performed, using three different techniques of regression estimation (OLS, MLE and SUR). Preceding the analysis, some diagnostic tests were carried out to determine whether the data set support some assumptions of the Least Square Regression, such as; normality, homoscedacity, and multi-colliniarity. Though some of the assumptions were violated, therefore the independent variables for the two models were transformed to ensure consistency of the estimation.

The research work has revealed that the Maximum Likelihood Estimator (MLE) has the minimum standard error for both the Gross Domestic Product (GDP) and External Reserve (ER) data, which determined the Maximum Likelihood Estimator (MLE) to be the most efficient among the three methods. Recommended, therefore, is the Maximum Likelihood Estimation which is capable of yielding a better estimate of Economic Growth.

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